Abstract

This paper presents a novel image space path planning method that takes mechanical constraints into consideration. The aim is to synthesize sub-goal images that can be used as reference inputs for image-based visual servoing systems. Synthesized images are constructed based on screw decompositions represented in a projective space. The projective representation allows image synthesis for three-dimensional objects to be made without any metric information or calibration processes. In the heart of the method, an orientation-generating operator is defined in the projective space to generate an image for an arbitrarily orientated object. The proposed scheme not only ensures field of view conditions by planning a straight path, but also provides physically realizable images of a properly oriented gripper to avoid mechanical constraints such as joint limits. Computer simulations are conducted to show the feasibility of the proposed scheme.

1. Introduction

Image-based visual servoing (IBVS) is a control scheme for vision-based robotics in which a task function is defined using image features that can be directly obtained from vision sensors [1]. Thanks to a direct feedback of image information, IBVS has superior advantages in that 3D model is not necessary and that the positioning precision does not seriously rely on the camera calibration compared to position-based visual servoing (PBVS) [2]. However, IBVS causes problematic results when the initial discrepancy between goal and current configurations is large [3]. The convergence is not guaranteed and also the feature points may leave the camera's field of view. It is mainly because that the relationship between image space and the controller input space is non-linear and highly coupled. Therefore, the image Jacobian, which is an essential part in IBVS, is not valid if the current configuration is not close enough to the goal configuration.

In this context, we previously proposed a novel method that resolves the mentioned problems by planning a straight path in the image space [4,5]. This method synthesizes a number of intermediate views of a robot gripper to construct the image trajectories that allow the robot gripper to track a straight path in a 3-D workspace. If two sets of image points corresponding to the initial and goal configurations are given within the image boundaries, the straight path is sufficient to ensure that the entire trajectories of the image points remain within the image boundaries as well. However, since those images are synthesized without consideration of mechanical constraints such as joint limits and kinematic singularities, the generated images may not be realizable even though they are right images of a 3-D object. Usually, the mechanical constraints are handled using the redundancy of given task [6]. However, the consideration of the mechanical constraints at the task planning level has not been actively studied.

In this paper, we extend our early work [4] by considering the mechanical constraints in generating image trajectories. Neither the Euclidean reconstruction nor pose estimation is involved with the help of a projective framework. The main goal is to synthesize images of virtual grippers whose orientations are properly adjusted so as not to encounter the mechanical limits while still holding a straight path strategy. To achieve this goal, an explicit representation for arbitrary orientations has to be developed first. Note that the orientations of all virtual grippers obtained in [4] are restricted by a single screw-axis direction. In other words, all allowable orientations have only one
degree of freedom: the angle about a fixed screw axis. We introduce an additional image of the gripper from a different viewpoint in order to explicitly represent arbitrary orientations. This introduction gives some additional parameters for the orientation of the gripper. With those parameters, we define an orientation-generating operator (OGO) within the projective framework. Determination of the parameters, which is equivalent to choosing a specific orientation, consists of three phases. First, the joint velocities that avoid mechanical limits are computed based on the gradient projection method (GPM). In this phase, image-point constraints are released to give enough degrees of redundancy in the image Jacobian. In the second phase, the corresponding image point velocities are computed back from the joint velocities using the forward image Jacobian for the entire points. At this time, the image Jacobian constrains full degrees of freedom so that all of the depending image variations can be found. Next, the parameters are found using the Jacobian of the OGO. The resulting projective points are projected onto the image plane to obtain the desired sub goal images.

In section 2, OGO is defined based on screw decomposition of projective displacement. Section 3 presents how to determine the parameters of OGO to avoid joint limits. Some simulation results are shown in section 4.

2. Orientation Generating Operator

In this section, we extend the idea of [4] to liberate the rotational motion from being restricted by a single screw axis. It is assumed that a projective transformation associated with a displacement can be computed from image-point correspondences.

If an additional set of image points is available such that the corresponding position of the object is different from the two given initial and goal positions, we can find two more projective displacements. This implies we can find two more sets of projective points representing the directions of two additional screw axes and their conjugated circular points. Suppose that the additional image points of the gripper are given and their correspondences with respect to the initial image points are known. The point correspondences allow us to compute the projective representations of those points and two additional projective transformations that relate the initial and goal gripper points. Let \( G_i^a \) be the projective coordinates of the \( i \)th gripper point corresponding to the additional image points. As depicted in Fig. 1, let \( H_2 \) and \( H_3 \) be the projective transformations from \( G_i \) to \( G_i^u \) and from \( G_i^a \) to \( G_i^u \), respectively, estimated using the method described in [4]. As shown in the case of \( H \), three projectively invariant points can be found for each projective transformation, \( H_2 \) or \( H_3 \), from its eigenvectors. Let \( \{E_2^p, E_3^p, E_4^p\} \) be the three linearly independent eigenvectors associated with eigenvalues \( \{1, e^{j\alpha}, e^{-j\alpha}\} \), where the subscripts \( p = 1, 2 \) and 3 correspond to the projective transformations \( H_2 \) and \( H_3 \), respectively. Then an orientation generating operator (OGO) with three parameters \( \theta_1, \theta_2 \) and \( \theta_3 \) can be defined as

\[
\Omega(\theta_1, \theta_2, \theta_3) = \prod_{p=1}^{3} E_p D(\theta_p) E_p^{-1} \quad (1)
\]

where

\[
E_p = \begin{bmatrix} G_1^{u*} & E_2^p & E_3^p & E_4^p \end{bmatrix}
\]

\[
D(\theta_p) = \text{diag}\{1, e^{j\theta_p}, e^{-j\theta_p}\}
\]

and \( G_1^{u*} \) is a projective point not belonging to the plane at infinity. The above definition can be best interpreted as a product of three projective transformations constructed by screw motion decompositions. More specifically, the transformations correspond to three consecutive rotational motions about the rotational axes \( G_1^{u*} E_2^t \), \( G_1^{u*} E_3^t \) and \( G_1^{u*} E_4^t \). We can generate an arbitrary orientation by adjusting the three \( \theta_1, \theta_2 \) and \( \theta_3 \), which is depicted in Fig. 2. One requested assumption is that the three directions of the screw axes should be linearly independent. In other words, the three direction vectors should not belong to one
onto the null space of $H$ and $H_2$ are not co-linear on the plane at infinity. The projective point $G_i^*$ can be considered as another parameter that defines the center of the rotational motions. Assume for a given $k$ that $G_i^*(i=1,\cdots,k)$ are projective coordinates of the points of $k$th virtual gripper to which the initial gripper points $G_i(i=1,\cdots)$ have to be transferred by a pure translation. The pure translation has to be performed such that $G_i^*$ corresponds to the reference point $G_i$ which is conveniently selected among the initial gripper points $G_i(i=1,\cdots)$. Pure translation of projective points is also described in [4].

The virtual gripper points $G_i^*(i=2,\cdots)$ are then transformed to new projective points $\tilde{G}_i^*(i=2,\cdots)$ by the OGO with certain parameter values in order to set a modified orientation about the center point $G_i^*$. If $k$ is assumed to be a time index of the given task, the parameter values at the final time $k=n$ have to be

$$
\begin{bmatrix}
\theta_1 \\
\theta_2 \\
\theta_3
\end{bmatrix}
= 
\begin{bmatrix}
\Theta_1 \\
0 \\
0
\end{bmatrix},
$$

where $\Theta_1$ is the principal angle associated with $H$. The remaining problem is then how to determine the intermediate values of the parameters such that they are continuously varied from $[0,0,0]^T$ to $[\Theta_1,0,0]^T$ to perform the given task while making the resulting orientations not violate the mechanical constraints. Note that $\theta_2$ and $\theta_3$ may have non-zero values not to encounter mechanical constraints during the task.

3. Determination of OGO parameters

If kinematic constraints such as joint limits are taken into account, joint space would be a better platform to represent robot configurations than Cartesian space. In this case, the image Jacobian should be defined as a mapping from the joint velocities to the image point velocities to perform a visual servoing task. For this reason, we assume that the image Jacobian can be estimated and updated online from the observations of the image points and control inputs, which are represented in joint space instead of Cartesian space. The methods of [7] can be used to estimate this new image Jacobian by simply replacing the notations related to the robot configuration.

In general, the joint-limit avoidance of a robot manipulator is implemented by defining a secondary task that utilizes task redundancy in such a way that it may not affect the primary task. The GPM is one such method widely used in the literature [8]. Suppose that the $2m \times n_q$ image Jacobian is given as

$$
\dot{s} = J_s \dot{q},
$$

where $\dot{s}$ is a $2m \times 1$ vector corresponding to the velocities of two dimensional coordinates of $m$ image points and $\dot{q}$ is $n_q \times 1$ vector consisting of the joint velocities. If the image feature points are selected to have redundancy in $J_s$, an infinite number of joint velocity vectors $\dot{q}$ exist for a given $\dot{s}$. Using GPM, the joint velocity vector that avoids joint limits can be computed as

$$
\dot{q} = J_s^* \dot{s} + (I - J_s^* J_s) \nabla O,
$$

where $O$ is a performance criterion related to the joint limits and $(I - J_s^* J_s) \nabla O$ is the homogeneous solution of (3). The homogeneous solution does not affect the primary task since the projection operator $(I - J_s^* J_s)$ projects the vector $\nabla O$ onto the null space of $J_s$. Note that (4) is no longer valid if $J_s$ has no redundancy to perform the secondary task.

In this paper, GPM is partly utilized to determine the orientations of the intermediate virtual grippers. First, a
primary task is defined to be a positioning task of a single point selected among the gripper points while a secondary task is defined to avoid the joint limits. Assume that \( G_i \) among the gripper points is chosen as a point object for the primary task. Then the goal point for the positioning task is dynamically selected among the knot points \( G^*_i (k = 1, \ldots, n) \) according to the current position. For example, if the point object \( G_i \) is currently located around the knot point \( G^*_i \), then the next primary task is to place the point object onto \( G^*_{(i+1)i} \) and the secondary task is to employ joint velocities that can avoid the joint limits without disrupting the primary task. In this step, the necessary image Jacobian, conveniently denoted by \( \tilde{J}_s \), consists of only two row vectors related to the two-dimensional image coordinates of the image point. If \( \tilde{J}_s \) is the full image Jacobian for the entire gripper points, \( \tilde{J}_s \) can be constructed with two row vectors of \( J_q \) such that they correspond to the image coordinates of \( G_i \). Since the dimension of the image Jacobian is 2×6 for a 6-DOF manipulator, the primary task is definitely a redundant task. For the secondary task, the performance criterion \( O(q) \) of [8] is adopted in this paper and given as below:

\[
O(q) = \sum_{i=1}^{n} \frac{(q_{i,max} - q_{i,min})^2}{4(q_{i,max} - q_{i})(q_{i} - q_{i,min})},
\]  

where \( q_i \) is the \( i \)th joint angle and \( q_{i,max} \) and \( q_{i,min} \) are the upper and the lower limits, respectively, on the joint angle \( q_i \). The resulting joint velocities \( \dot{q} \), computed from (4) by using \( \tilde{J}_s \) instead of \( J_q \), allow the gripper to rotate spherically with a fixed center point \( G^*_{(i+1)i} \). The direction of the rotation complies with joint limit avoidance. The corresponding variations of the image points or the image point velocities can be computed back by applying the resulting joint velocities to (3). At this time, the image Jacobian contains all rows for the entire image point variations of the gripper. The image point velocities provide a direction in the image space to which the gripper has to move to avoid the joint limits. However, this direction may not comply with the final goal configuration, which can be represented with OGO parameters as (2) at final center point \( G^*_i \). These two objectives have to be incorporated together in such a way that the gripper moves smoothly and surely from the initial to goal configuration while avoiding the joint limits during the movement. For this purpose, the image point velocities are transformed into OGO parameter velocities. Since the projective points for the gripper in a modified orientation can be computed using OGO and the corresponding image points can be obtained by projecting the projective points with the projection matrix \( P_i \), the image point velocities are related to the OGO parameter velocities as

\[
\dot{s} = J_\Omega \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{\theta}_5 \\ \dot{\theta}_6 \end{bmatrix} = J_\Omega \dot{\theta}_s ,
\]

where

\[
J_\Omega = \partial P_\Omega (\theta_1, \theta_2, \theta_3) G^*_i .
\]

\( P_i \) can be arbitrarily chosen to get a specific projective displacement [4]. Therefore, the OGO parameter velocities for joint limit avoidance can be derived from the image point velocities \( \dot{s} \) as follows:

\[
\dot{\theta}_s = J_\Omega \dot{s} .
\]

For the final goal configuration, on the other hand, the OGO parameter variations should be made according to the final and current values of the parameters. Since the final values of the parameters should be \([\Theta_1, 0, 0]^T \), the parameter velocities are determined as

\[
\dot{\theta}_s = K \begin{bmatrix} \Theta_1 \\ 0 \\ 0 \\ \Theta_2 \\ \Theta_3 \end{bmatrix}.
\]

To get the overall parameter velocities, \( \dot{\theta}_s \) and \( \dot{\theta}_s \) are combined in mutually exclusive manner so as not to conflict with each other. One such combination can be realized by using the performance criterion in (5). The following combination is used in this paper.

\[
\dot{\theta} = \zeta \dot{\theta}_s + (1-\zeta) \dot{\theta}_s .
\]

where

\[
\zeta = 1 - \frac{1}{1 + O(q)} .
\]

Note that if the joint angles are close to the joint limits, or equivalently, if \( O(q) \) has very large value, \( \dot{\theta}_s \) has a dominant effect. Otherwise, \( \dot{\theta}_s \) has a dominant effect. The resulting parameter velocities allow us to determine the next desired gripper points in the projective space. The
desired image points of the gripper are then obtained by projecting the projective points with the projection matrix \( P_1 \). If these sub goal images are given to the IBVS system with a conventional control law, the final goal can be achieved while avoiding the joint limits.

### 4. Simulations

Computer simulations have been performed to show the validity of the joint limit avoidance on a PC platform using Matlab toolbox. Fig. 3 shows the environment for the simulations. A six DOF manipulator of a PUMA-560 kinematic model is given and an artificial gripper is attached to it. It is assumed that a camera observes the gripper in a fixed position using simple pinhole camera model. Most of the conditions for simulations are the same as [4] except an assumption that an additional set of image points of the gripper is given.

Fig. 4 shows top view of the 3-D environment where the manipulator is in its initial position and the goal position is located at (0.2,0.3). The objective is to approach the goal position straight not to leave the camera's field of view. In this case, the screw axis associated with the displacement between the initial and goal gripper is parallel to the Z-axis.

Fig. 5(a) and Fig. 6(a) show the camera views of the manipulator performing visual servoing by using the resulting image trajectories generated without and with consideration of the joint limits, respectively. The corresponding side views of the 3-D environment captured in the middle of the servoing tasks are depicted in Fig. 5(b) and Fig. 6(b), respectively. It is shown in Fig. 5(b) that the gripper is facing down, which causes the second and third links move close to their joint limits. The rotated gripper about the axis parallel to Z-axis is still facing down since the initial gripper is also facing down. Fig. 6(b) shows that the links are more released as a result of the joint limit consideration.

The joint trajectories for the second and third joints are depicted in Fig. 7. The solid and the dashed lines show the results with and without consideration of the joint limits, respectively. The horizontal dashed lines indicate the joint
limits. It is clearly shown that the joints move away from their limits when the joint limits are considered in the generation of the image trajectories.

5. Conclusion

In this paper, a novel method has been proposed to increase the feasibility of the image-space path generation approach by taking the mechanical constraints into account. The generated images can be best used as reference inputs to the image-based visual servoing system that has large pose discrepancy. A particular advantage to this approach is that it does not necessitate any metric information or precise camera calibration. It has been shown via simulations that three views of gripper are enough to generate a view of an arbitrarily oriented rigid gripper without any 3-D information.

6. References


